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Radiative Corrections in the Angular Correlation of Monopole Pairs from O^{16} at Small Angles*

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The angular correlation of electron-positron pair emission in the $0^+ \rightarrow 0^+$ transition in O^{16} was investigated for very small angles. An enhancement of several percent in the emission rate at these small angles is expected, due to first-order radiative corrections represented principally by the vertex graph. The rapid rise in the correlation at small angles was experimentally verified, and the results are in agreement with the theory.

I. INTRODUCTION

The emission of electron-positron pairs in the nuclear $0^+ \rightarrow 0^+$ transition in O^{16} is perhaps the simplest electromagnetic process involving electrons and positrons. This is so because the energy of the transition is sufficiently high and the charge of the O^{16} nucleus is sufficiently low so that pair particles can be described to a very good approximation by plane waves. To first order in the fine structure constant the detailed structure of the pair emission—its dependence on angle and energy partition—is then essentially given by the simple quantity $I = |(u(p_+),\ u(p_-)|^2$, where $u(p_+)$, $u(p_-)$ are plane wave spinors for positrons and electrons, and the brackets indicate a scalar product.

If the polarization of the particles is not measured, I has to be summed over all spin directions; this gives a distribution function

$$S(W_+,\theta)dW_+d(\cos\theta) = (W_+W_- - 1 + p_+p_- \cos\theta)p_+p_-dW_+d(\cos\theta)\,, \quad (1)$$

$$W_+ + W_- = k;\quad k = 11.84;$$

here W_+, W_- are the energies of the positron and the electron and p_+, p_- the respective momenta in natural units. The quantity θ is the angle between the two particles. This equation was first verified experimentally by Devons and Lindsey[1] and will be referred to as the "Dirac formula."

The great simplicity of the pair emission process, coupled with the considerable amount of structure which it nevertheless possesses, makes it an interesting tool for investigating higher order radiative corrections to electromagnetic phenomena. A detailed analysis of first-order radiative processes in the monopole pair emission was carried out by Dalitz.[2] The process investigated in particular in the present work is the modification of the angular correlation by the interaction of a positron and an electron emitted with small relative velocity. This is the interaction represented by the "vertex graph" (Fig. 1). A measurement of this radiative correction is of interest because the electrons and positrons involved in this particular process have higher momenta than those encountered in the "classical" measurements of radiative corrections like the Lamb shift or the modification of the gyromagnetic ratio of the electron.

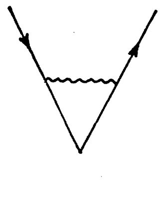
The predicted effect is large enough to be measured. At an angle of 3.5° and in the actual experimental conditions, the increase in intensity is about 4% with respect to the Dirac formula.

II. APPARATUS

The apparatus employed in these measurements is illustrated in Figs. 2 and 3. The pair particles were detected in two telescopes each consisting of an aluminum absorber for the scattered protons, a thin plastic scintillator (α,β) which determines the solid angle (in these two elements the electrons lose about 150 keV of their energy), and a large plastic scintillator (a,b) in which the energy of the particles is determined. The sum of the pulse heights in a and b, gated by a quadruple coincidence $\alpha + a + \beta + b$, was recorded in a multichannel analyzer. A representative spectrum is shown in Fig. 4. Most counts are seen to be concentrated in a peak corresponding to 5.03 MeV. This is the transition energy minus the masses of the two pair particles.

The discrimination level for pulses from a and b was set to record only pulses corresponding to electrons

FIG. 1. Vertex graph.



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[1] S. Devons and G. R. Lindsey, Nature **164**, 539 (1949).

[2] R. H. Dalitz, Proc. Roy. Soc. (London) **A206**, 521 (1951).

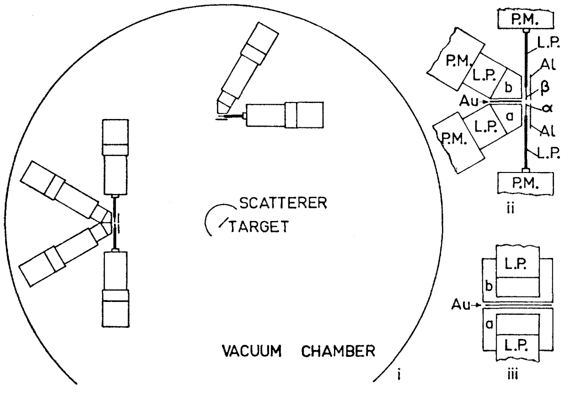

FIG. 2. Schematic drawing of apparatus. (i) The arrangement of the counters in the vacuum chamber. The moving telescope (b,β) is shown at the smallest and largest angles. The scatterer (Al 0.5 mm) was used only in the control experiment. (ii) Top view of the telescopes in the 3°40′ position: the thin scintillators (α,β), the thick scintillators (a,b) and the light pipes (L.P.). Aluminum absorbers of 0.05 mm in front of the thin scintillators stopped scattered protons and alpha particles. Gold foils of total thickness of 0.6 mm between the scintillators a and b absorbed more than 1.8 MeV of the energy of electrons passing from one scintillator into the other. (iii) Front view of the telescopes at 3.40°. The scintillators a, b extend beyond α and β in order to reduce end effects in the energy determination of the electrons.

emitted with kinetic energy of at least 1.5 MeV (allowing for the energy loss in α, β). In this way only the portion 1.5–3.5 MeV of the spectrum was recorded. It was found advisable to concentrate on such a relatively narrow band around the mean energy of 2.5 MeV because the spurious effects described below are smaller, and because the effect investigated increases with decreasing relative velocity of the pair particles and is noticeable only for electron pairs of nearly equal energy (and with a small angles between them).

The solid angle of each telescope was 2.7×10^{-4}, and the range of angles between the center lines of the telescopes was 3° 40′ to 180°. Measurements were carried out only up to 91° 30′.

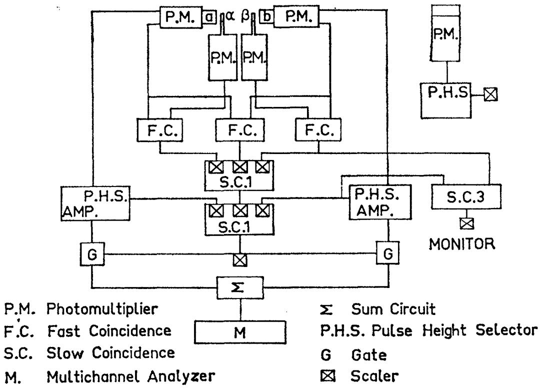

FIG. 3. Circuit diagram. The resolving times of the fast and slow coincidence circuits are, respectively,

$$2\tau_F=6\times10^{-9}\text{ sec.}\quad 2\tau_S=6\times10^{-7}\text{ sec.}$$

The slow coincidence S.C. 3 served as normalization.

III. MEASUREMENTS

The excited 0^+ state in O^{16} was produced in the reaction $F^{19}(p,\alpha)O^{16}$ with protons of 1.880 MeV from a 2-MeV Van de Graaff accelerator. This bombarding energy was chosen because it was found that at this energy the yield of pairs is relatively large compared to the yield of gamma rays from the other levels in O^{16} and in particular from the 3^- level at 6.13 MeV which lies very close to the 0^- level.[3] The target was made of CaF_2 of a thickness of about 90 keV for 873-keV protons and was prepared by evaporation on a 0.27 mg/cm² aluminum backing. The angular correlation was measured in a plane perpendicular to the proton beam at angles measured to be 3° 40′, 5° 20′, 7° 20′, 10° 20′, 20° 20′, 30° 30′, 61° 00′, and 91° 30′. The angle was changed every 20 min, and about 30 separate runs were carried out for each angle. A spectrum similar to the one shown in Fig. 4 was recorded for each run. Only the counts in the full energy peak were used in determining the correlation. The number of these counts was determined by integrating each spectrum over the range shown by the brackets in Fig. 4. To check the operation of the circuit counts were recorded with the aid of scalers at the various points indicated in the circuit diagram (Fig. 3). The measurements were normalized to the number of counts above the discrimination level in one of the telescopes.

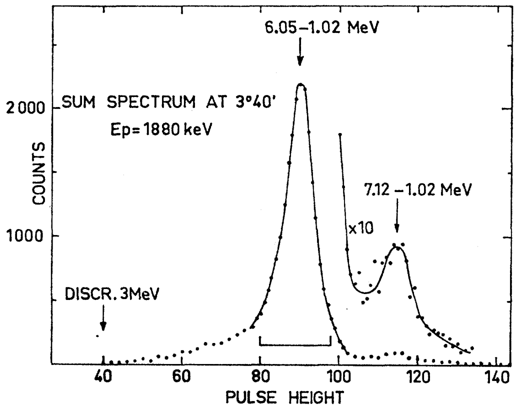

FIG. 4. Spectrum of the sum of energies recorded in the two telescopes at 3°40′. The energy resolution is about 10% and the peaks corresponding to monopole pairs and to dipole pairs are well separated. The energy discrimination (including absorption) corresponds to 3 MeV in the energy sum. The channels used for the correlation measurement are indicated by a bracket.

The measurements were corrected for a number of spurious effects which will now be described in detail.

A. Conversion Pairs

Apart from the monopole pair emission there is also a relatively weak emission of internal conversion pairs

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[3] S. Gorodetzky, G. Sutter, P. Chevallier, F. Scheibling, and R. Armbruster, Compt. Rend. **250**, 1028 (1960).

from the gamma-ray transitions. In particular, the pairs emitted in the $3^- \rightarrow 0^+$ transition (called π_6) cannot be resolved from the monopole pairs and are recorded in the same energy peak. Likewise the pairs emitted in the $2^+ \rightarrow 0^+$ and $1^- \rightarrow 0^+$ transitions (called π_7) can contribute by their low-energy tail to the peak of the monopolar pairs. The contribution of these two phenomena was evaluated by means of a series of measurements at angles of 3° 40′ and 91° 30″ at the resonance $E_p = 2030$ keV. At this resonance the intensity of the $0^+ \rightarrow 0^+$ transition is diminished by a factor of around 10 as compared to the $3^- \rightarrow 0^+$, $2^+ \rightarrow 0^+$ and $1^- \rightarrow 0^+$ transitions.

Moreover, a measurement using a three-crystal external pair spectrometer at 90° to the beam direction has verified that the relative intensity of these three transitions is the same at $E_p = 1880$ keV as at $E_p = 2030$ keV. In particular, the proportion of gamma rays from the 6.13 MeV level ($3^- \rightarrow 0^+$) is $15 \pm 2\%$. This ratio is in good agreement with measurements carried out by Lars Ask[4] at somewhat different proton energies.

The contribution of π_6 and π_7 pairs was determined by a second completely independent method. For this a separate angular correlation measurement was carried out at a proton energy of 340 keV. At this energy the 0^+ state is not excited, and all the observed pairs are produced in the $3^- \rightarrow 0^+$ transition.[5] The number of conversion pairs per count in a fixed 5-in.×4-in. NaI scintillator is thus determined. From this number and the intensity ratio of the gamma rays, the number of pairs at each angle at $E_p = 1880$ keV can be determined separately[6] for the transitions $3^- \rightarrow 0^+$ and $1^- \rightarrow 0^+$.

The number of dipole π_7 pairs can also be determined directly from the original angular correlation measurement because they give rise to a separate small peak in the sum spectrum (Fig. 4). The number of π_7 pairs determined in this way is consistent with the measurement described above and agrees well with the theoretical correlation.

The total correction including both the octupole pairs (from the $3^- \rightarrow 0^+$ transition) and the low-energy tail from the dipole pairs is 1.5% of the number of coincidence counts at 3° 40′ and decreases with increasing angle.

B. "Cross Talk" between the Telescopes

As the effect considered in this work is a very sharp function of angle and is measurable practically only at angles close to 0°, one has to eliminate very carefully all events which are propagated directly from one telescope to the other. The problem is aggravated by the fact that the solid angle subtended by the counters at the source is very small and the number of "true" coincidence events per count in one telescope is therefore also small, whereas the spurious coincidences (per telescope count) are a function only of the distance between the telescopes and do not depend on the distance of the counters from the source.

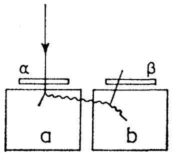

FIG. 5. Events which may give rise to spurious coincidence counts.

The most serious spurious effects of this type would be caused by the transfer of particles from one large crystal to the other. This was eliminated by placing gold foils of total thickness 0.6 mm between the counters. Although the gold is not thick enough to stop all electrons completely, it absorbs a sufficient fraction of the total energy ($E \geqslant 1.8$ MeV) so that electrons so transferred cannot be part of a "full energy event." One would, therefore, expect some spurious effects of this nature to manifest themselves in the low-energy tail but not in the full energy peak which is used in the correlation measurement.

Counts can nevertheless be transferred by means of bremsstrahlung photons which are produced in one crystal, enter the other crystal and produce an electron in it. Two effects due to events propagated by bremsstrahlung were found to be the most probable:

(i) This is shown schematically in Fig. 5: A particle of close to the maximum energy enters one telescope and emits a bremsstrahlung photon in the large scintillator. This photon enters the other large scintillator and produces in it an electron with an energy close to the photon energy. The electron loses most of its energy in the large scintillator but escapes and reaches the thin scintillator, thus producing a fourfold coincidence of about the right energy sum.

(ii) This is shown schematically in Fig. 6: Here too a bremsstrahlung photon emitted in one large scintillator [say, (a)] produces an electron in the other large scintillator (b); however no fourfold coincidence is actuated by this particle. Fourfold coincidence are produced by genuine pairs, one member traversing β and stopping in b as usual. The number of coincidence counts is therefore not changed by the bremsstrahlung transfer. However, the spectral distribution is changed and the total effect may be that a count, which would normally (i.e., at large angles) be registered in the

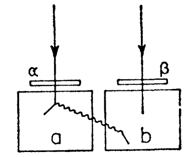

FIG. 6. Events which may give rise to spurious coincidence counts.

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[4] Lars Ask, Arkiv Fysik 19, 219 (1961).

[5] The correlation obtained is in agreement with the theoretical correlation for an $E3$ transition.

[6] G. Goldring, Proc. Phys. Soc. (London) 73, 556 (1959).

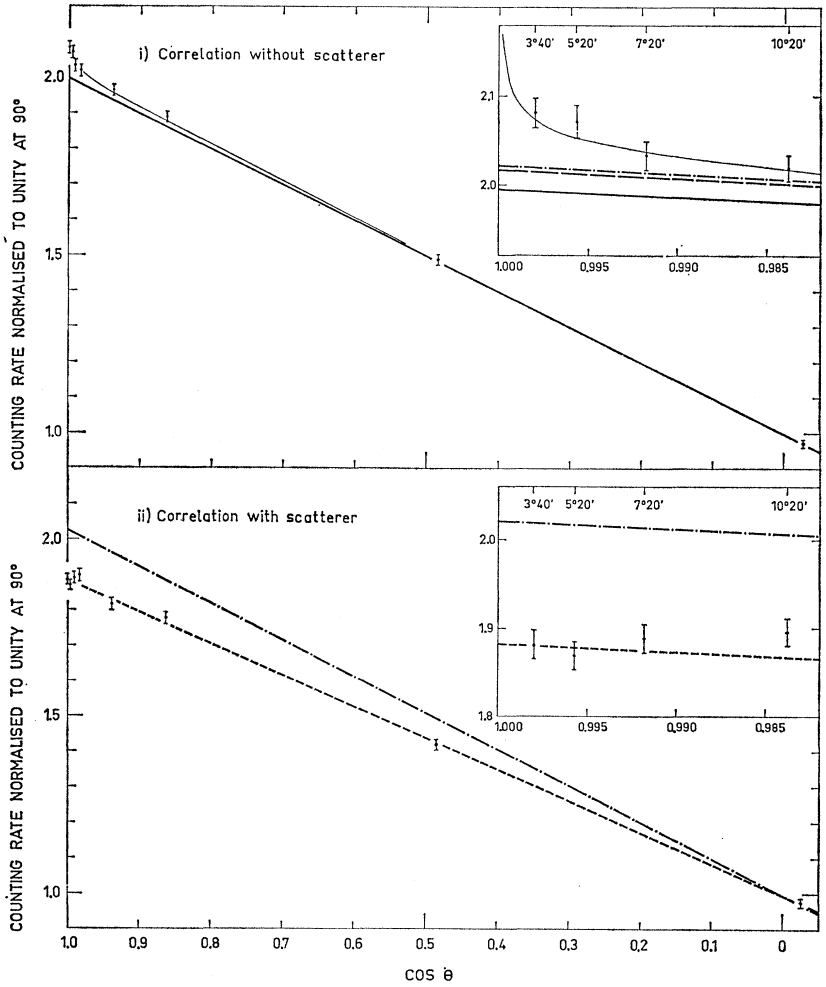

FIG. 7. Angular correlation of the monopole pairs. (i) The upper figure shows the measured values (corrected as explained in the text), the Dirac straight line (heavy line), and the angular correlation function which includes radiative corrections (light line) (corrected for the finite aperture of the counters). These are all normalized to unity at 90°. The enlarged drawing shows the small angle feature and includes the straight line fit to theoretical (dashed line) and experimental (dash-dot line) points at the four angles $\geqslant 20°$. (ii) The lower figure shows the correlation with an aluminum scatterer and the straight line fit (dashed line) to points at the four angles $\geqslant 20°$ (normalized to unity at 90°). An attenuation of 13.6% in the slope of this line appears, due to the diffusion effect (as compared to a 13% attenuation calculated with allowance for the absorber thickness and the energy spectrum of the electrons).

low-energy tail of the sum spectrum, is transferred to the full energy peak and is illegitimately counted. Similarly, the annihilation gamma rays of positrons can transfer counts to the high energy peak and also out of it.

Both these transfer effects were examined directly in suitable coincidence and absorber arrangements, and they were found to contribute less than 0.2% of the total coincidence counts at 3° 40′ decreasing rapidly with increasing angle. It was, therefore, not necessary to correct for these effects. The smallness of their contributions is explained by the relatively high energy discrimination and is very well confirmed by the behavior at small angles of the correlation measured with a scatterer placed between the target and the counters in the manner indicated below.

C. Random Coincidences and General Background

Corrections for random coincidences between two telescope counts (having the right energy sum) have also been introduced, these were essentially independent of angle. The general background (due presumably to cosmic radiation) is almost negligible in magnitude.

IV. RESULTS

The angular correlation corrected in this fashion is shown in Table I and in Fig. 7. The abscissa of the figure is in units of $\cos\theta$ and the Dirac formula (1) yields a straight line on this scale (the "Dirac line"):

$$f(\theta)=C(1+a\cos\theta).$$

The integration of (1) over $w_+(4\leqslant w_+\leqslant k/2)$ and over the angular range of the telescopes gives $a=0.9970$. The line computed by integrating the complete expression given by Dalitz[2] has also been drawn. The normalization is $f(\pi/2)=1$. The experimental points are seen to fit the Dalitz curve very well. At the small angles the points lie definitely above the Dirac line and the increments, we conclude, represent the radiative corrections. At the four larger angles ($\geqslant 20°$) a least-square fit to the experimental points gives a slope a

TABLE I. Experimental and theoretical correlations normalized to unity at 90°.

θ	3° 40′	5° 20′	7° 20′	10° 20′	20° 20′	30° 30′	61°	91° 30′
Dirac	1.995	1.993	1.989	1.981	1.935	1.860	1.483	0.974
Dalitz	2.073	2.051	2.035	2.015	1.957	1.876	1.488	0.974
Experimental results	2.081±0.018	2.070±0.018	2.033±0.017	2.019±0.016	1.965±0.015	1.880±0.015	1.488±0.014	0.974±0.012

$=1.024\pm0.017$ which is larger than that of the Dirac line ($a=0.9970$). In fact, the contribution of the radiative corrections is not negligible at these angles. The angular correlation is only approximately a straight line, and a least-square fit to the four theoretical points 20°, 30°, 60°, and 90° gives a slope $a=1.0193$.

In order to ascertain that the increased counting rates at small angles are not due to some unsuspected geometric effect, the measurements were repeated with an aluminum ring 0.5 mm thick placed around the target. The particles on their way to the counters were, thus, scattered in the aluminum and the angular correlation is accordingly "smeared out." A delta function would be spread over about 30° by this scattering, and we would therefore expect the small-angle feature of the pair correlation to be practically destroyed. The results of this measurement are also shown in Fig. 7. A straight line was here drawn with a least-square fit to the four points at angles greater than 20°, and it will be noticed that the small angle points lie quite close to this line as expected. This proves that the sharp small-angle feature observed in the pair correlation is definitely a property of the pair particles *as they leave the target.*

The coefficient a for the correlation measured with scatterer is found to be: $a=0.885\pm0.016$.[7] The reduction in a is of course also due to the smearing-out effect of the scatterer, and indeed if the reduction in a is calculated on the basis of the well-known formula for the multiple scattering of electrons, the result (13%) is quite consistent with the observed decrease in a (13.6%).

V. CONCLUSION

In Fig. 7 we show the first-order radiative corrections to the angular correlation at small angles. As was mentioned before, the most important contribution in this region comes from the vertex graph and represents essentially the interaction of electrons and positrons with low relative velocity.

The measured points are seen to fit the Dalitz line very well. If, to express this agreement quantitatively, we assume the angular correlation of the radiative correction terms to be given correctly by the theory and use the experimental results merely to determine the "strength" of the interaction, and if we express this strength as an effective fine structure constant, then we get from our results:

$$\alpha_{\text{eff}}=\alpha(1.11\pm0.20).$$

ACKNOWLEDGMENT

We would like to express our thanks to U. Maors for programming and carrying out the computations on the WEIZAC electronic computer.

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[7] Note that the straight line fit to the eight experimental points gives a slope $a=0.891$.